Newton Raphson :

Example

Find The only real root of The eqn:

$$f(x) = x^3 - x - 1 = \text{Zero}$$

Ans by Hand: ينبغى معرفة أول رقم يتم تعويضه فى المعادلة كالآتى

say $x=1 \longrightarrow$ if $f(1) = -1$

say $x=2 \longrightarrow$ if $f(2) = 5$

$\longrightarrow \therefore$ Value of x ranges from 1 To 2 To give $f(x) = \text{Zero}$

$\therefore$ Assume: ~~f(x)...~~ initial value of $x = 1.5$

$$f(x) = x^3 - x - 1 = 0$$

$$f'(x) = 3x^2 - 1$$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

$x_0 = 1.5$: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.3478260$

$x_1 = 1.347826$: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.32520039$

$x_2 = 1.32520$: $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \underline{\underline{1.324718}}174$

$x_3 = 1.324718174$: $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = \underline{\underline{1.324717}}95$

stop at values of x in The Last Two step are equal

$$x_3 = x_4 = 1.3247$$

$$\therefore \quad x = 1.32471 \quad \#\#\#$$

Newton-Raphson Example

Aim

To demonstrate how the Newton-Raphson method works.

Learning Outcomes

At the end of this section you will:

- Understand how the Newton-Raphson method works,
- Be able to apply the Newton-Raphson method to certain problems.

Example

Use Newton's Method to find the only real root of the equation $x^3 - x - 1 = 0$ correct to 9 decimal places.

We have $f(x) = x^3 - x - 1$ and $f'(x) = 3x^2 - 1$. Since $f(1) = -1$ and $f(2) = 5$, the function has a root in the interval $[1, 2]$ since the function changes sign between $[1, 2]$. Let us make an initial guess $x_0 = 1.5$.

Newton's formula here is

$$x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1} = \frac{2x_n^3 + 1}{3x_n^2 - 1},$$

so, with our value of $x_0 = 1.5$, our approximation for x_1 is given by

$$x_1 = \frac{2(1.5)^3 + 1}{3(1.5)^2 - 1} \approx 1.3478260....$$

Using a scientific calculator, it is possible to finish the sum. Take the value of x_1 and repeat the above calculations using this as the initial guess. The resulting answer will be x_2. Again repeat the procedure until the 9^{th} decimal place remains unchanged.

n	x_n	$f(x_n)$
0	1.5	0.875
1	1.34782608696..	0.100682173091..
2	1.32520039895..	0.002058361917..
3	1.32471817400..	0.000000924378..
4	1.32471795724..	0.000000000000..
5	1.32471795724..	

Therefore $r = 1.324717957$ correctly rounded to 9 decimal places.

Exercise

Find the root of the function $y = x^3 + 4x^2 + 7$ in the vicinity of $x = -4$ correct to 5 decimal places.

Related Reading

Stroud, K.A. 2001. *Engineering Mathematics.* 5^{th} Edition. PALGRAVE.

Anton, H., I. Bivens, S. Davis. 2005. *Calculus.* 8^{th} Edition. John Wiley & Sons.